

# Small Two-Dimensional Surface Excrescences on Aircraft Wings Approaching Separation

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When studying the performance of aircraft wings at high-lift configurations it is important to appreciate the effects that discontinuities in the surface can cause in promoting premature flow separation. Having caused two-dimensional incompressible fluid flow to separate from the floor of a specially-designed wind tunnel in an experimental research program, the authors introduced small two-dimensional surface excrescences on the wind-tunnel floor, some distance upstream of this flow separation region. The techniques used to establish that the clean-surface flow was separating, to measure its gross characteristics, and then to make the necessary measurements when the excrescences were present, are described. Provided that the excrescences were small, having  $y^+$  values that were less than 500, and were as far upstream of the separation region as at least 14 excrescence heights, the investigation that the downstream separation process was unaffected by their upstream presence. An appropriate lag-entrainment integral boundary-layer-prediction technique supported this conclusion.

## Nomenclature

$C_D \equiv 2$ Excrescence Drag/ $\rho h U_e^2$	= excrescence drag coefficient per unit span
$c$	= chord length of aerofoil
$c_f$	= wall skin friction coefficient
$G \equiv (H - 1)/H(2/C_f)^{1/2}$	= Clauser's defect shape parameter
$H \equiv \delta^*/\theta$	= boundary-layer shape factor
$h$	= characteristic dimension (height or depth) of a surface excrescence
$h^+ \equiv hu_\tau/\nu$	= excrescence Reynolds number; dimensionless roughness height
$J \equiv (H - 1)/H$	= boundary-layer shape-factor parameter
$m \equiv (x/U_e)(dU_e/dx)$	= equilibrium boundary-layer velocity-gradient parameter
$p$	= freestream pressure
$Re_\theta$	= local Reynolds number, in terms of boundary-layer momentum thickness
$U \equiv U(y)$	= streamwise mean velocity component
$U_e$	= freestream velocity
$u$	= velocity fluctuation from the mean
$u_\tau \equiv U_e(c_f/2)^{1/2}$	= friction velocity
$x$	= surface coordinate in the freestream direction
$y$	= distance measured normal to the floor, from the working-section floor
$y^+ \equiv yu_\tau/\nu$	= Reynolds number expressed in terms of $y$ ; inner boundary-layer variable

$\beta \equiv \frac{\delta^*}{\tau_w} \frac{dp}{dx}$	= Clauser's pressure gradient parameter
$\gamma$	= flow intermittency factor, i.e., fraction of time the flow at a given location remains turbulent
$\delta$	= boundary-layer physical thickness
$\delta^* \equiv \int_0^\infty [1 - (U/U_e)] dy$	= boundary-layer displacement thickness
$\theta \equiv \int_0^\infty U/U_e [1 - (U/U_e)] dy$	= boundary-layer momentum thickness
$\mu$	= fluid dynamic, or absolute, viscosity
$\nu \equiv \mu/\rho$	= fluid kinematic viscosity
$\rho$	= fluid density
$\tau_w$	= wall shear-stress
<b>Subscripts</b>	
$o$	= at zero external pressure gradient
$l$	= distance downstream, measured from the excrescence location
$R$	= flow reattachment length, measured from separation location

## Introduction

ON the wing of a high-lift aircraft the otherwise smooth surface could be marred by the presence of localized discontinuities and irregularities, termed surface excrescences. Some of these excrescences, such as Pitot tubes and antennae, are required to accomplish specific technical functions; some, such as those formed by ice, arise naturally while others, such as those left when leading-edge slats and trailing-edge flaps retract into the main aerofoil, are formed during manufacture. Although these latter excrescences are reduced to a minimum, their total removal is often uneconomic. Young and Paterson<sup>1</sup> have described the effects that surface excrescences can have on the maximum lift generation from a wing, stating that in regions of strong adverse pressure gradient the increase they

Received Dec. 13, 1990; revision received Oct. 15, 1991; accepted for publication Oct. 29, 1991. Copyright © 1991 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

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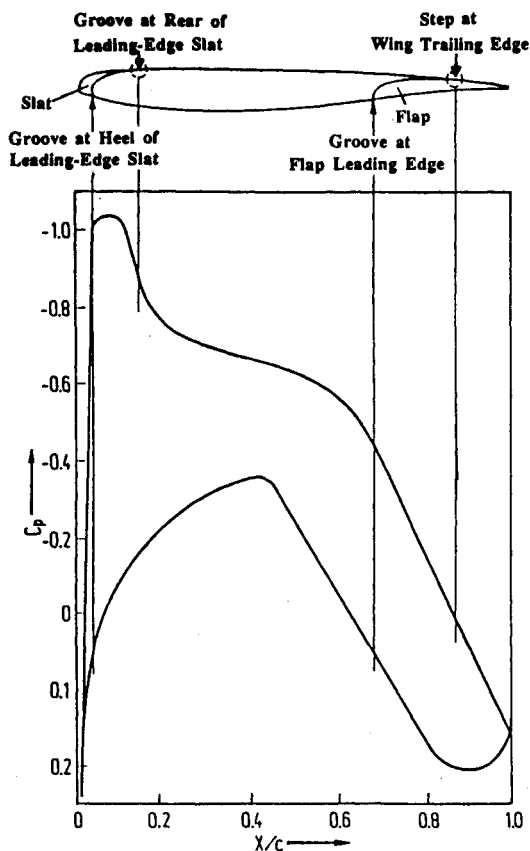


Fig. 1 Surface excrescences formed during wing manufacture under both strong favorable and strong adverse pressure gradients.

cause in the boundary-layer momentum thickness can trigger or hasten flow separation. However, Nigim, and Cockrell<sup>2</sup> concluded that when a flow is about to separate from a wing, the increase in boundary-layer thickness resulting from an excrescence upstream and continuing downstream to the vicinity of the separation region did not significantly trigger the separation process.

A typical pressure distribution around a wing at high-lift with presumed locations of two-dimensional surface excrescences, i.e., excrescences that extend across the flow with their axes perpendicular to the flow direction, is shown in Fig. 1. Both excrescences that are formed during manufacture, and those naturally-formed through ice deposition, are often in strongly adverse or strongly favorable pressure gradient regions. Though the increment in boundary-layer momentum thickness caused by such excrescences, is carried out into the boundary layer downstream, the question this investigation addresses is; if in their absence, this downstream flow is already about to separate from the surface, will this separation tendency be significantly enhanced by their presence? To answer it, an experimental and a computational study were recently conducted at Leicester University and these are described in this article.

## Experimental Study

### Apparatus

In this work, incompressible flow approaching separation was first established in the absence of any surface excrescences on the working-section floor of a specially designed low-speed boundary-layer wind tunnel. This wind tunnel was of a closed-return design, with a working section that was 1.14-m wide, 0.85-m high, and 4.9-m long. In the working section, the nominal speed range was 0–30 m/s, and the turbulence intensity was less than 0.5%. The working-section roof, shown in Fig. 2, was constructed from a reinforced flexible sheet of 3-mm-thick plywood, its location being maintained by 16 elec-

trically-controlled scissor jacks installed along its length. By variation of the spacing between the working section floor and roof, a wide range of favorable, neutral, and adverse pressure gradients could be imposed on the flat lower instrumented surface of the working section. In the strongly adverse pressure gradient boundary-layers that were developed in this study, the most useful measurement technique for streamwise mean velocities, and for turbulence intensities, was pulsed-wire anemometry, originally described by Bradbury and Castro<sup>3</sup> and in the present context discussed elsewhere by Alhusein and Cockrell.<sup>4</sup> Wall shear-stress measurements up to flow separation were also made by pulsed-wire anemometry, using an appropriate sensing probe.

### Establishing Flows that Approach Separation

Following Clauser<sup>5</sup> and Mellor and Gibson,<sup>6</sup> by representing the relationship between the freestream incompressible velocity  $U_e$  through the working section and the streamwise distance  $x$  as

$$U_e \propto x^m$$

so that

$$m = \frac{x}{U_e} \frac{dU_e}{dx}$$

where the exponent  $m$  is necessarily negative, equilibrium turbulent boundary layers that approach separation were established in the wind-tunnel working section. Townsend<sup>7</sup> showed that for values of  $m$  having a modulus less than 0.2, the same external conditions over a surface can produce two different types of boundary-layer flow. Later, East, Smith, and Merryman<sup>8</sup> demonstrated this conclusion experimentally, showing that both flows were possible for all negative values of  $m$ ; one for which the boundary layer was attached (thin and the shape factor  $H$  was relatively small) and the other for which the boundary layer could separate from the surface, (thicker than the former boundary layer) and for which the shape factor  $H$  was much greater than for the other flow. The relationship they established for  $H$  as a function of the exponent  $m$  is shown in Fig. 3. In this figure, the shape factor parameter  $J$ , where

$$J = (H - 1)/H$$

is also shown.

In common with the shape factor  $H$  from which it is derived, the shape factor parameter  $J$  is a function of the flow Reynolds number. In strongly adverse pressure gradients, it is more appropriate to use than either  $H$  or Clauser's universal defect shape parameter  $G$ , where

$$G = \frac{H - 1}{H} \left( \frac{2}{c_f} \right)^{1/2} = J \left( \frac{2}{c_f} \right)^{1/2}$$

for with the approach of flow separation,  $J$  is contained within an acceptable range from 0 to 1 for a variation in both  $H$  and  $G$ , from 1.0 to infinity.

When the flexible roof was set to establish a freestream velocity variation with an appropriate value of  $m$ , e.g.,  $m = -0.18$ , the asymmetric diverging shape of the working section ensured that both types of boundary-layer flow were established, the attached flow on the flat instrumented floor, and the separating flow from the roof. To re-energize this latter flow over the roof, and thus obtain flow separation from the floor, some passive forms of boundary-layer control were employed. To inhibit the flow separation delta-shaped vortex, generators were attached to both the roof and the side walls. These were supplemented by a slot formed across the roof, located 0.5 m downstream from the maximum suction region. Since the working-section pressure was subatmospheric, air

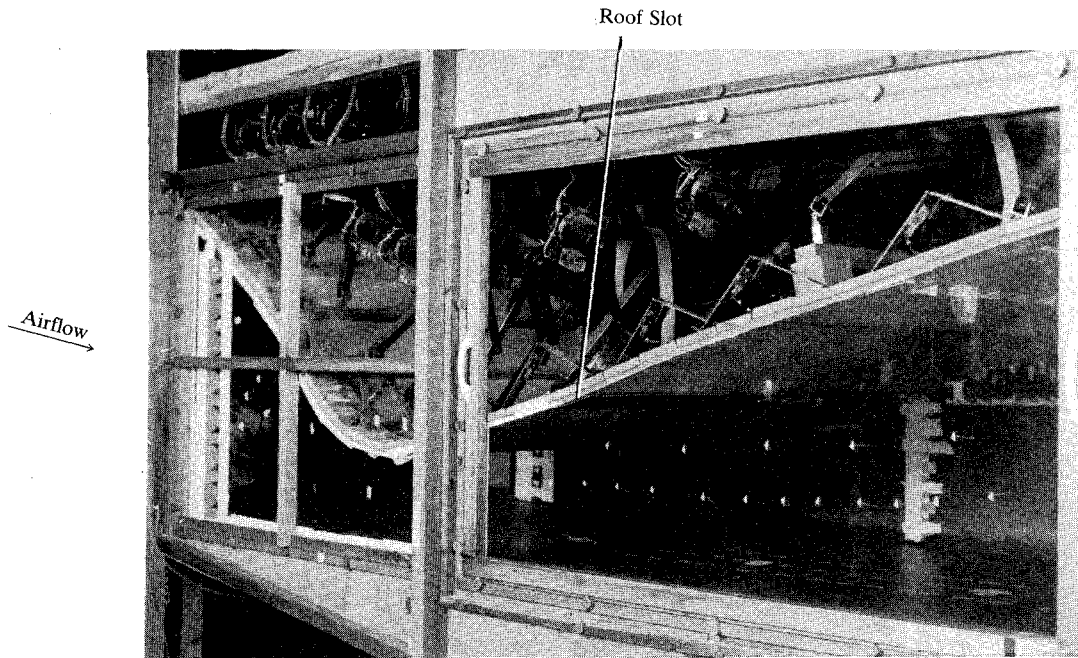


Fig. 2 Working section roof set to provide an adverse pressure gradient over the floor.

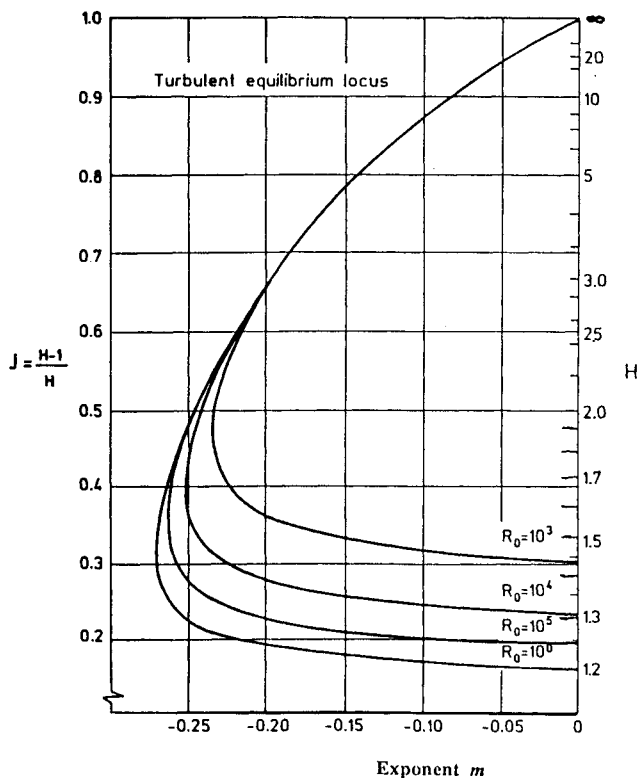


Fig. 3 Shape factor  $J = (H - 1)/H$  and shape factor  $H$  as functions of flow parameter  $m$  for equilibrium turbulent boundary layers.<sup>8</sup>

was drawn through this slot, into the working section and downstream over the working section roof. Both the vortex generators and the slot can be seen in Fig. 2, where wool-tuft tracers are also visible. As separation conditions were approached, the establishment of near two-dimensional flow in the working section was a largely empirical technique, and the tracers, that were permanently attached to the working-section walls, played an important part in ensuring that the required conditions were achieved.

Flow separation did not take place at a readily identified line across the working section, but within a region, and the location of this region varied. In Fig. 4 a sequence of smoke

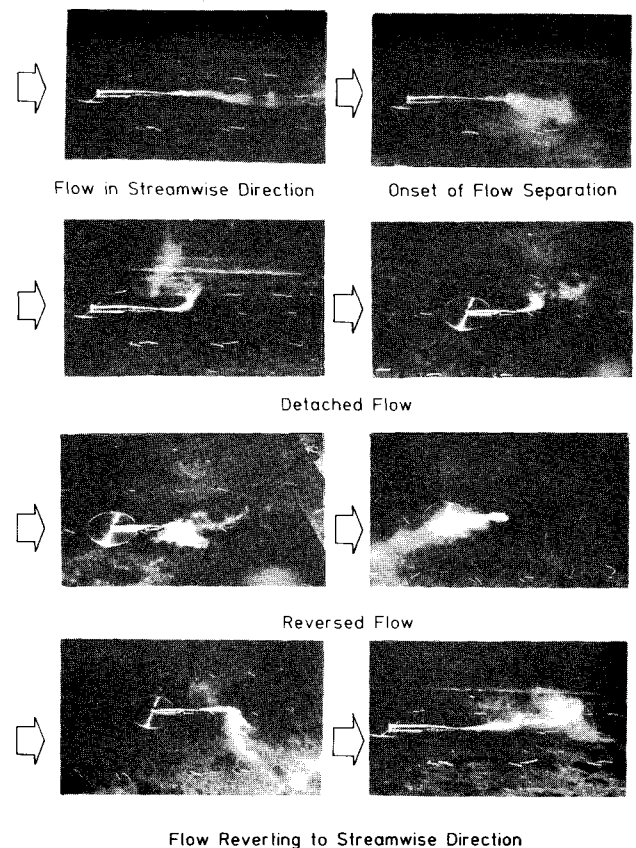


Fig. 4 Sequence of smoke photographs showing unsteady separated flow from a location on the working-section floor.

photographs is shown showing, by their variation with time, the region of flow separation from the surface at a distance of about 4 m downstream from the working-section inlet. Three-dimensionality dominated this separation region.

The freestream velocity distribution through the working section, determined by pulsed-wire anemometry, is shown in Fig. 5 where the subscript  $o$  denotes the location where the pressure gradient is zero. An adverse pressure gradient was established over the working-section floor from  $x = 1.365$  m

to  $x = 4.650$  m. The maximum freestream velocity was approximately 16 m/s, the Reynolds number per unit length where the measurements were made was  $1.05 \times 10^6$ . Under these conditions, the displacement and momentum thicknesses of the boundary layer,  $\delta^*$  and  $\theta$  respectively, together with the shape factor  $H$ , were as shown in Fig. 6.

As the flow approached separation an intermittent flow-reversal occurred close to the wall. Downstream of this region the mean external pressure gradient decreased sharply. The separation point was defined as the location where the time-

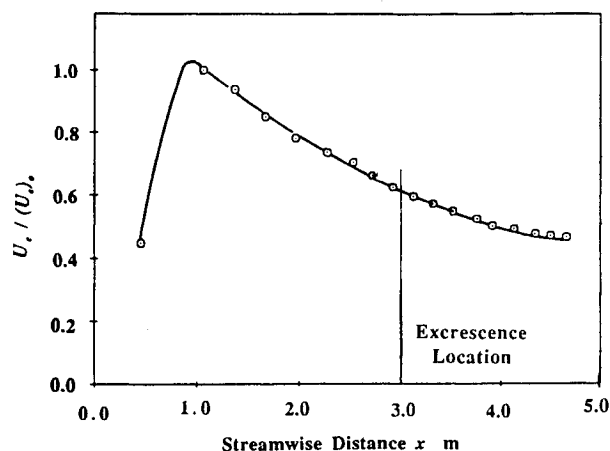


Fig. 5 Freestream velocity distribution through the wind-tunnel working section.

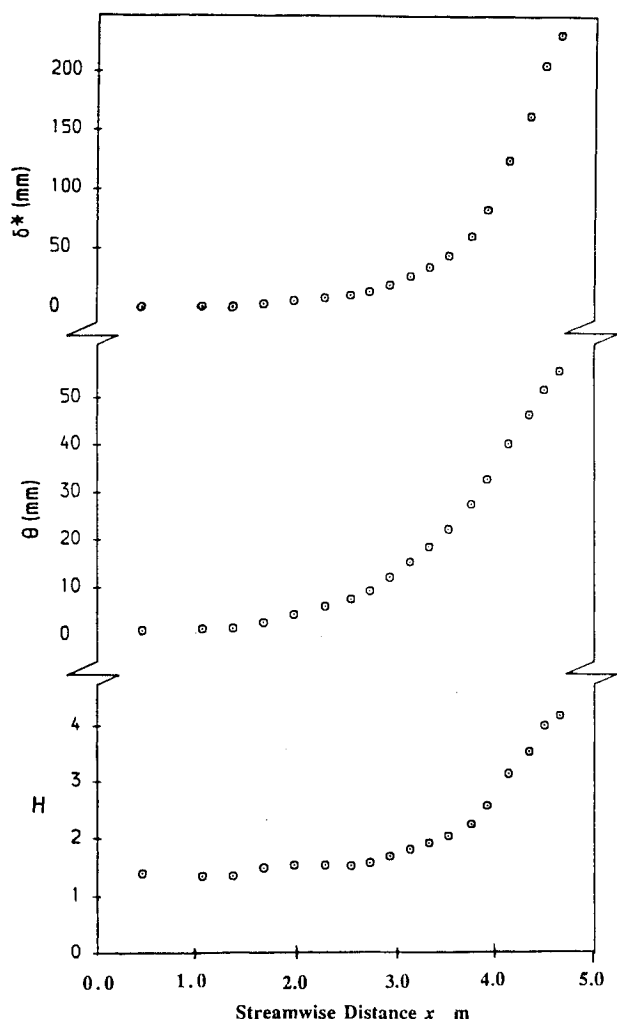


Fig. 6 Thickness of the boundary layer developed over the working-section floor.

averaged wall skin friction coefficient  $c_f$  equalled zero, or where the intermittency factor  $\gamma$ , defined as the fraction of time whereby the flow at a point in the boundary layer remained turbulent, was equal to 0.5.

Three different techniques were used to measure the wall skin friction coefficient. All of these depend, in different ways, on the turbulent boundary-layer structure. They were: by use of a Preston tube with Patel's<sup>9</sup> correlation; by employing the Ludwig and Tillmann<sup>10</sup> empirical relationship, that

$$c_f = 0.246 Re_\theta^{-0.268} 10^{-0.678H}$$

where  $Re_\theta$  is the local Reynolds number determined in terms of the boundary-layer momentum thickness  $\theta$ ; and also by the previously-mentioned pulsed wall-shear-stress probe. Results that these techniques gave are shown in Fig. 7. The data reveal the characteristic sharp decrease in  $c_f$  as separation is approached and indicate that flow separation occurred in the region of  $x = 4.0$  m. Results compare well with those in Fig. 8 that were obtained using the pulsed-wire probe over 5-min periods to determine at what location the intermittency factor  $\gamma$  was equal to 0.5.

Comparisons were made between the flow in the working section and other strongly adverse pressure-gradient flows that Simpson et al.<sup>11,12</sup> and Nigim<sup>13</sup> (flow 5) established. The adversity of the freestream pressure gradient is denoted by Clauser's<sup>5,14</sup> pressure-gradient parameter  $\beta \equiv (\delta^*/\tau_w)(dp/dx)$  where  $\tau_w$  is the wall shear stress. Values of  $\beta$  are shown plotted against streamwise distance  $x$  in Fig. 9.

Sandborn and Kline<sup>15</sup> demonstrated that intermittent flow separation commences at a value of  $J$  that is dependent on

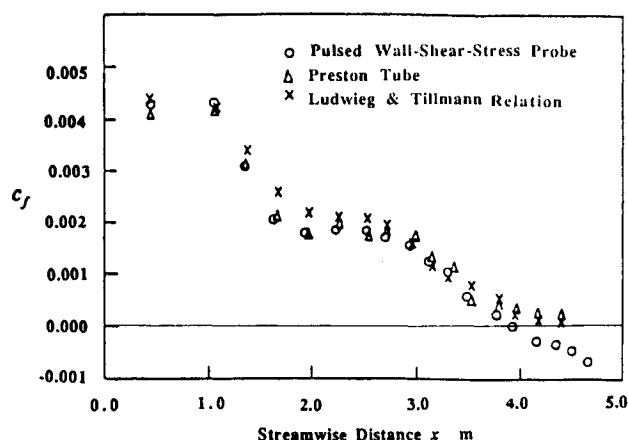


Fig. 7 Skin friction coefficient  $c_f$  measured along the working-section floor.

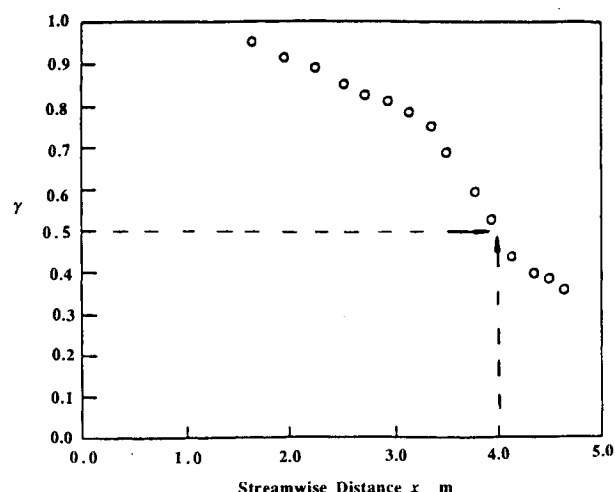


Fig. 8 Variation of the intermittency factor  $\gamma$  with distance measured along the working-section floor.

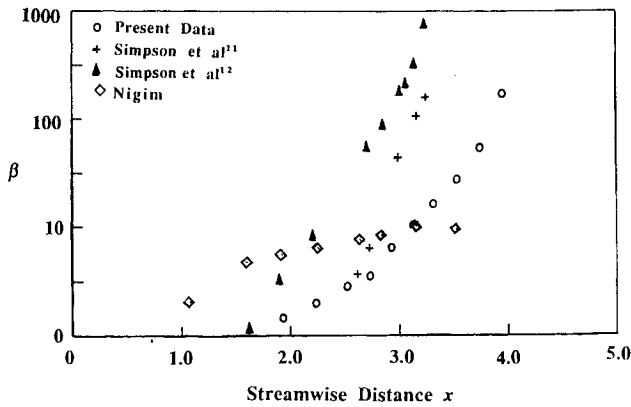


Fig. 9 Variation of Clauser's pressure gradient parameter  $\beta$  with streamwise distance  $x$  for a number of adverse pressure gradient flows.

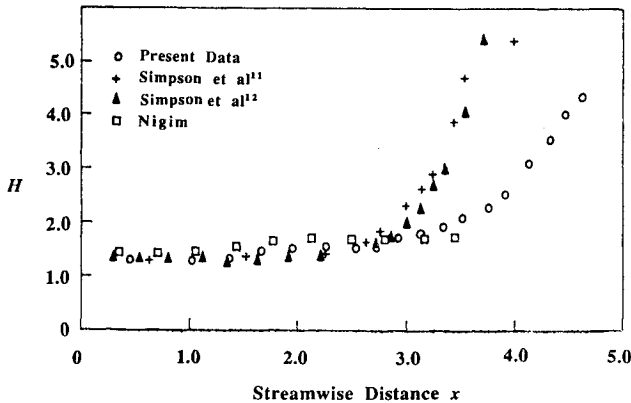


Fig. 10 Shape factor  $H$  variation with streamwise distance  $x$  for the flows of Fig. 9.

the ratio  $\delta^*/\delta$ . As separation is approached, typical values of  $J$  are between 0.52 and 0.60 thus, corresponding values of the shape factor  $H = 1/(1 - J)$  are from 2.1 to 2.5. Figure 10 shows the shape factor  $H$  plotted against streamwise distance  $x$  for the flows considered in Fig. 9. For the present data, where separation is deemed to have occurred at  $x = 4.0$  m, the corresponding shape factor  $H$  equals 2.5, a value Simpson achieved in a shorter streamwise distance of about 3.1 m but was not achieved at all over the streamwise distance considered in Nigim's adverse pressure gradient flow 5.

#### Surface Excrescences

For this study, the chosen two-dimensional surface excrescences were square cross-section ridges extended across the flow for the full width of the working section. Their shape was chosen to combine the function of a forward-facing step with that of an immediately following backward-facing step. Four different sizes of ridges were used; 10-mm, 19-mm, 25-mm, and 50-mm square. These ridges were fixed on the working-section floor at a common location, 3.0 m downstream from the working section inlet, in the adverse pressure gradient flow, as shown in Fig. 5. In the absence of an excrescence at the chosen location, the physical boundary layer thickness  $\delta$  was about 100 mm, with these ridge sizes it was possible to vary  $h/\delta$ , where  $h$  is the height of the excrescence, from 0.1 to 0.5. With the exception of the largest ridge, the ridge excrescences were small in the sense that they lay within the law of the wall region of the turbulent boundary layer, for in their absence at the chosen location, the inner variable  $y^+ = yu_\tau/\nu$  (where  $u_\tau$  is the friction velocity) was about 500, whereas their dimensionless roughness heights  $h^+ = hu_\tau/\nu$  were 180, 350, 450, and 900, respectively. Although they lay within a strongly-adverse freestream pressure gradient, the drag coefficient  $C_D$  for these ridge excrescences was well-represented in terms of the local clean surface skin friction coefficient  $c_f$

by Gaudet and Johnson's<sup>16</sup> zero pressure gradient relationship

$$C_D/c_f = 150 \log_{10} h^+ - 190$$

the universality of this latter relationship following the characteristics of the logarithmic region of the boundary layer where the three smaller excrescences and most of the larger excrescence were immersed.

#### Flow Downstream of the Surface Excrescences

Mean velocity and normal Reynolds stress profiles were obtained, as shown in Figs. 11 and 12, by traversing the pulsed-wire probe through the boundary-layer downstream of the surface excrescences. In these figures the distance  $x_i$  is measured downstream from the location of the excrescences. The profiles display typical features of a reattached boundary layer in that the inner part quickly relaxes to the shape of the undisturbed boundary-layer profile, while the outer part, that is more strongly affected by longer-living large turbulent eddies, lags behind. Figure 11 shows that near the midpoint of the boundary layer, the rapid increase in the inner-part velocity, and the slower response of the outer-part velocity, results in an appreciable difference in the velocity gradient.

Little difference was detected in the Reynolds stress profiles obtained downstream of the three smaller surface excrescences. Since they did not differ significantly from the clean-surface Reynolds stress measurements, only one set of profiles has been presented in Fig. 12. This lack of change to the turbulence structure accords with Bradshaw and Wong's<sup>17</sup> observation that small excrescences upstream cause only weak perturbations to the velocity boundary-layer structure, nec-

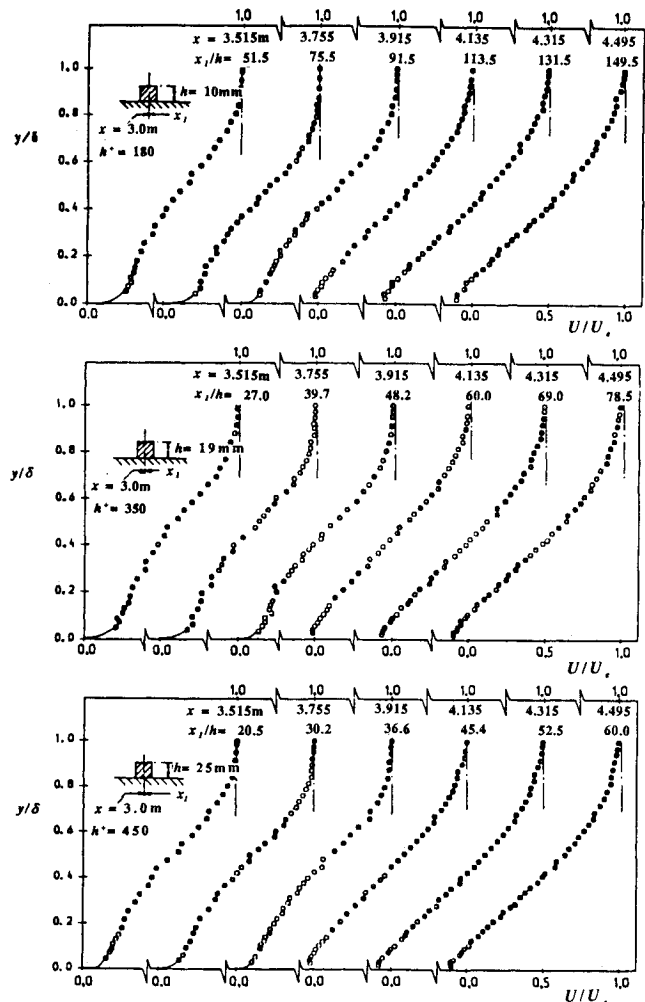


Fig. 11 Mean velocity profiles measured downstream of small surface excrescences.

essarily increasing the boundary-layer momentum thickness downstream but not modifying the essential velocity profile shape.

Effects caused by the surface excrescences to the gross boundary-layer characteristics  $\theta$ ,  $H$ , and  $c_f$  of the downstream flow are shown in Figs. 13a, 13b, and 13c. At a given streamwise location,  $x$  immediately downstream of the excrescence, the boundary-layer momentum thickness must increase as the excrescence height is increased, for the momentum thickness  $\theta$  is a measure of the drag on the surface. In the flow separation region immediately downstream of the excrescence, the shape factor  $H$  also increases, decreasing again before the anticipated increase takes place further downstream. As Fig. 13 shows, far downstream, the velocity boundary layer is similar in shape to that measured in the absence of the excrescence. Since the shape factor  $H$  there is little changed by a surface excrescence, the increasing momentum thickness  $\theta$  that occurs must be accompanied by a corresponding increase in the boundary-layer displacement thickness  $\delta^*$ . Consequent on flow separation immediately downstream of an excrescence, the local skin friction coefficient  $c_f$  decreases significantly, but after about 14 excrescence heights downstream, boundary-layer reattachment results in a recovery value of the order of that in the absence of the excrescence. For the tests conducted on small excrescences in adverse pressure gradients, the reattachment length  $x_R$  approximated to  $14h$ .

Proceeding downstream from the excrescence, a distance  $x$  of some 40 excrescence heights, apart from the increment

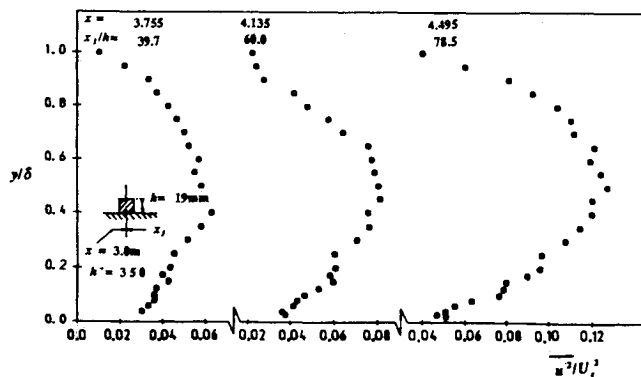


Fig. 12 Streamwise Reynolds normal stress  $\overline{u^2}/U_e^2$  measured downstream of small surface excrescences.

in momentum thickness that the excrescence directly causes, the other gross boundary-layer characteristics display surprisingly little evidence of the excrescence upstream. Both Mueller and Robertson<sup>18</sup> and Plate,<sup>19</sup> examining zero free-stream pressure gradient flows, reported a similar result. Since the Ludwig and Tillmann relationship for  $c_f$  indicates that the local skin friction coefficient is a function of both  $\theta$  and  $H$ , even at such a distance downstream of the excrescence some skin friction variation might be anticipated. However, in the present flow separation study, even in the absence of the upstream excrescence, the local skin friction coefficient in this region was very small. When the excrescence was present, further reduction in skin friction coefficient was only 2% of the clean-surface value. As this reduction lay within the uncertainty of the experimental measurements that were made, such a reduction was considered to be negligible.

To examine if the response of the flow downstream was strongly dependent on whether the excrescence height was greater or less than the local logarithmic boundary-layer thickness, the larger 50-mm square ridge was also used as a surface excrescence. Under the conditions tested, the dimensionless roughness height  $h^+$  of this ridge was nearly twice that of local logarithmic boundary-layer  $y^+$ . Although much of its surface was influenced by the near-wall boundary-layer characteristics, Fig. 14a makes it clear that the skin friction coefficient  $c_f$  downstream showed no propensity to recover within the reattachment length  $x_R$  of 14 excrescence heights (0.7 m), as is demonstrated in Fig. 13c for the smaller surface excrescences. The lack of reattachment downstream of this larger ridge is also implied by Fig. 14b, where over the whole downstream region, the intermittency factor  $\gamma$ , as measured with the pulsed-wire anemometer, is less than 0.5.

### Computational Study

Using the lag-entrainment integral boundary-layer prediction technique originally developed by Green, Weeks, and Brooman<sup>20</sup> and described elsewhere by Cockrell, Nigim, and Alhusein,<sup>21</sup> the presence of surface excrescences was modeled by localized step changes in boundary-layer displacement thickness  $\delta^*$ , momentum thickness  $\theta$ , and in the skin friction coefficient  $c_f$ . Both the separation region immediately downstream of the excrescence, and the boundary layer that developed following the subsequent reattachment, were well modeled by this computational technique. The comparisons

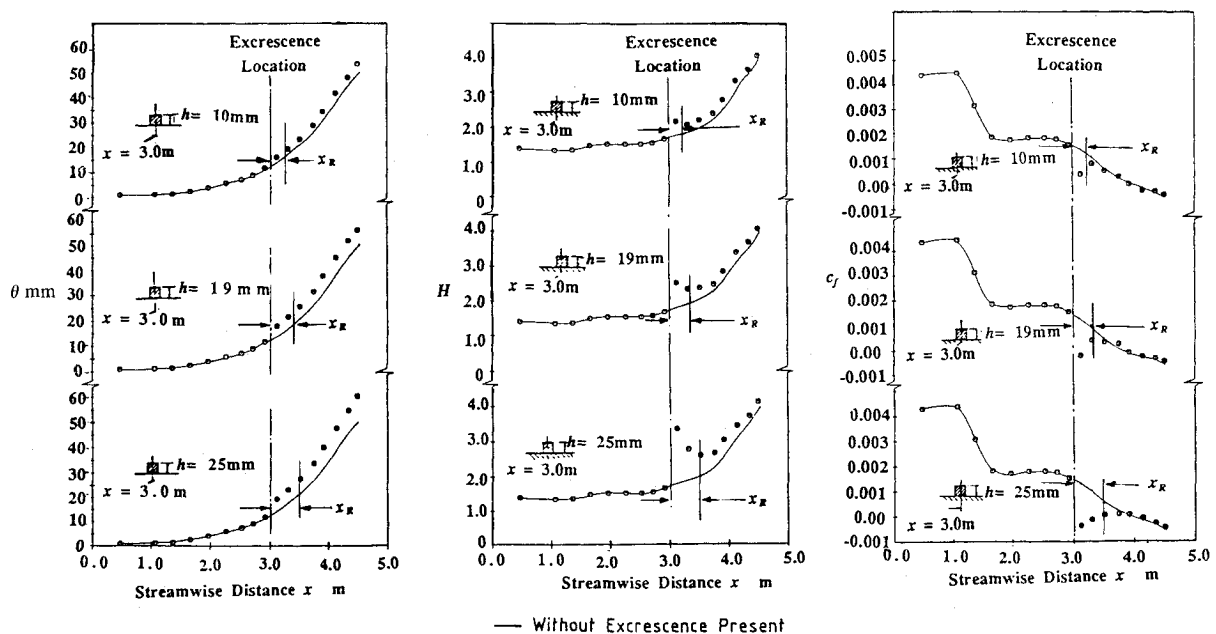


Fig. 13 Effects of the surface excrescences on the gross characteristics of the downstream-developed-boundary layer.

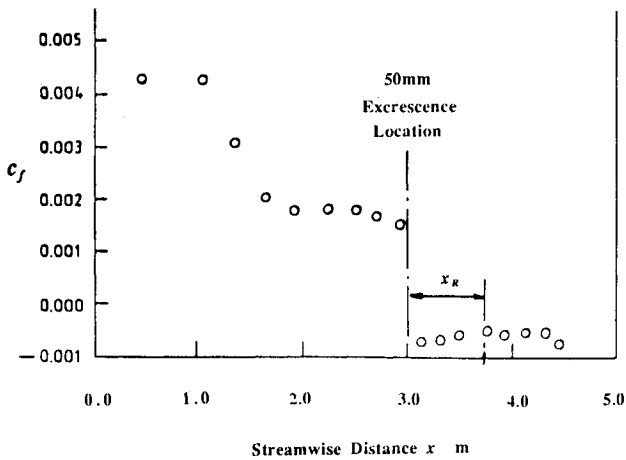
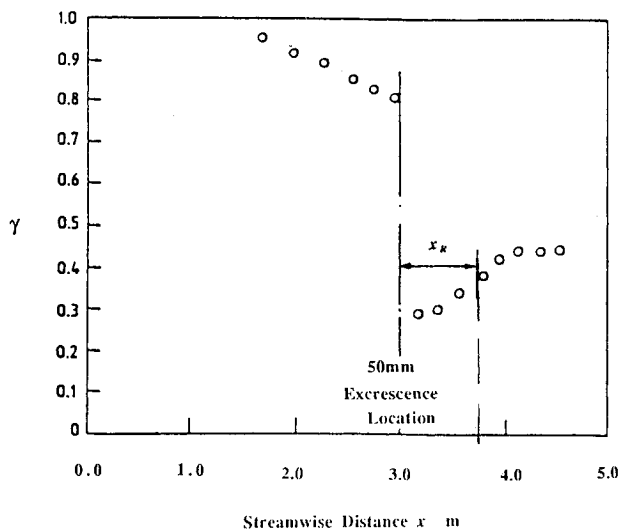
A—Variation in Skin Friction Coefficient  $c_f$ B—Variation in Intermittency Factor  $\gamma$ 

Fig. 14 Flow downstream of the 50-mm excessence.

between the experiments conducted and these computer predictions are shown in Fig. 15 and are seen to be very satisfactory.

### Discussion

The conditions observed, where a boundary layer approaching separation is only to a negligible extent, affected by the presence of a two-dimensional excessence upstream are:

1) The excessence must be far enough upstream of the clean-surface flow separation region for the flow to reattach again at its rear. For small excessences this condition is satisfied in at least 14 excessence heights, regardless of the freestream pressure gradient;

2) The dimensionless roughness height  $h^+$  of the excessence must be small enough to lie within the thickness of the undisturbed logarithmic layer at its location. Under these conditions, the freestream pressure gradient has little effect on the excessence drag, the downstream separation region, and the subsequent flow reattachment. Possibly, the flow downstream would also be unaffected by the presence of larger surface excessences, however, when such an excessence was tested it was found to significantly affect the flow downstream, where the separation bubble that developed immediately downstream extended to include the clean-surface flow separation region further downstream.

In the downstream separation region, the boundary-layer momentum and displacement thicknesses are increased by the presence of any small surface excessence upstream, but otherwise, the boundary-layer is essentially unchanged from that with a clean surface (there is no change in the relationship for its skin friction coefficient  $c_f$  in terms of  $\theta$  and  $H$ ). Thus, any increase in skin friction caused by the excessence is proportional to the local skin friction developed in its absence and as clean-surface separation is approached, the effect that any small surface excessence upstream can have on the skin friction downstream, is necessarily negligible.

With larger surface excessences, not only is their drag coefficient dependent on the freestream pressure gradient, but flow characteristics downstream are greatly changed by their presence. For these reasons, two-dimensional surface excessences extending above the logarithmic layer, significantly affect the boundary layer downstream and might well influence flow separation.

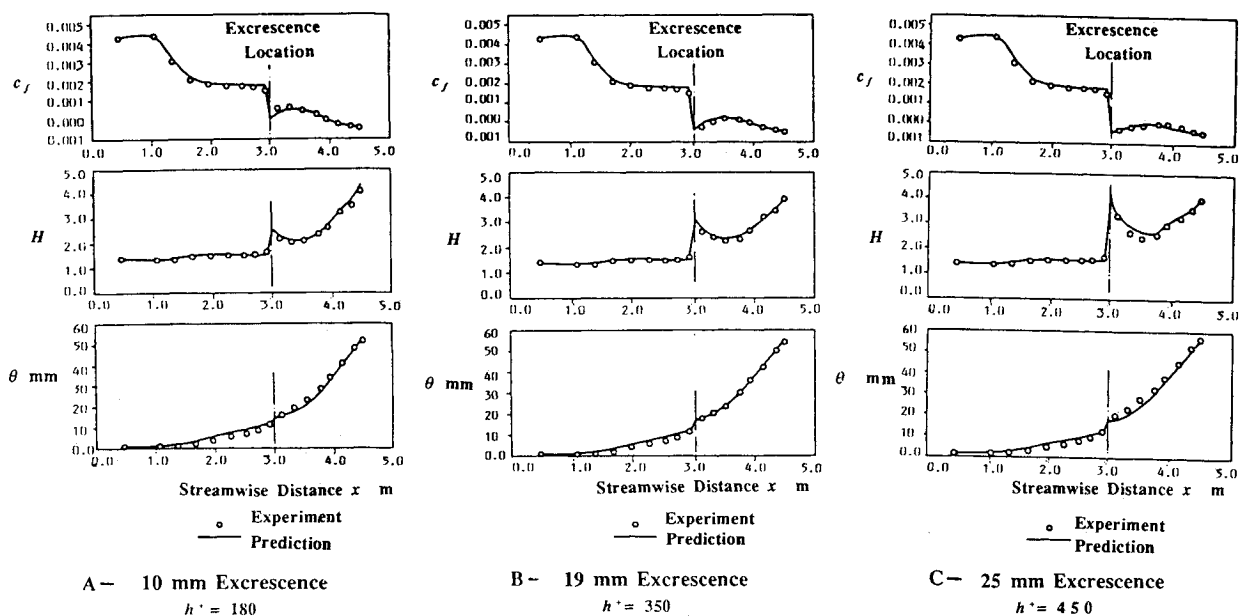


Fig. 15 Flows downstream of the small excessences; comparison between experiments and prediction.

## Conclusions

A strongly-adverse pressure gradient leading to separated flow such as that found on an aircraft wing in a high-lift configuration, was established in a specially designed low-speed boundary-layer wind tunnel. This flow possessed comparable characteristics with those developed by other experimenters. Two-dimensional surface excrescences, representative of those that occur during the wing manufacture or by ice deposition during flight, were set in the adverse pressure gradient at a location well upstream of the flow separation region. Provided that these excrescences lay within the law of the wall boundary-layer region, they were found to have a negligible effect on the provocation of flow separation downstream.

Good agreement was obtained between these experimental results and those obtained from an appropriate integral lag-entrainment boundary-layer prediction technique used to compute the flow downstream of the surface excrescence.

## Acknowledgment

The authors gratefully acknowledge the support of Mu'tah University, Jordan by which this research was made possible.

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